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Method for the making of digital Nyquist filters with null inter-symbol interference and corresponding filtering device

The field of the invention is that of filtering of digital signals. More specifically, the invention relates to the making of Nyquist filters such as those used for example in baseband and single-carrier transmission.

Nyquist filters play an essential role in transmission systems. In most cases, this filtering is distributed between sending and reception in the form of two filters known as Nyquist root filters. When sending, the aim especially is to limit the band of the signal sent. At reception, the filter must eliminate all the different noises that may damage the useful signal, especially the interference noise due to adjacent channels. In modern systems, these filters are made in digital form. There are many methods to synthesize these filters. A key element of this synthesis, apart from compliance with frequency specifications, is the need to obtain the smallest possible inter-symbol interference (ISI).

To ensure the most efficient possible conditions of transmission, it is sought generally to obtain real coefficient filters which, ideally, will meet the following criteria:

- null inter-symbol interference;
- frequency selectivity;
- sending and reception filters forming matched pairs;
- phase linearity.

Appendix 1 describes these different aspects and details and the corresponding constraints. This appendix, just like the following ones, is of course an integral part of the present description.

A complete transmission system must generally also carry out other indispensable functions such as:

- analog filtering at sending which is designed to reject the output harmonics from the digital filter as well as analog filtering at reception whose function is to limit the spectrum of the signal to be sampled solely to the useful band;
- digital-analog conversions (DAC) at sending, and analog-digital conversions (ADC) at reception.

The invention essentially relates to the obtaining of digital filters, and no detailed description shall be given of the functions relating to the analog part. It must be noted however that, in practice, the analog filtering to be implemented is substantially less restrictive in terms of steepness of the filters than digital filtering. More specifically, the complexity of the embodiment of the rejection filter at sending is directly related to the value of the oversampling factor chosen.

As for the converters, the most notable fact from the viewpoint of filtering is that of digital-analog conversion which introduces a sine(x)/x filtering that may be conventionally compensated for by the analog filtering. This processing can also be done digitally.

Furthermore, for reasons of implementation, the filters are generally sampled at double or quadruple frequency.

Figure 1 illustrates the digital part of a communications system of this kind. The signal to be sent X(z) 11 supplies the sender 12 which delivers a filtered signal Y(z) 13 transmitted through a transmission channel 14 to a receiver 15 that delivers the output signal S(z) 16. The signal X(z) 11 is first of all subjected to oversampling 121 and then to a filtering 122 $F_t(z)$. In the receiver, the signal undergoes a reception filtering $F_r(z)$ 151 and then a decimation 152. With T as the elementary delay linked to the sending and reception digital filters, we have T_s = MT, where T_s is the symbol duration and M the factor of oversampling.

The value of having a high oversampling factor is especially the fact that it facilitates the making of the analog filter that will follow. One drawback is that it correspondingly increases the speed of the converter (DAC). In practice, the most reasonable compromise corresponds to a value of M = 2 or 4. The invention relates specifically to the case where the oversampling factor is M = 4.

Nyquist filter construction techniques are already known. However, the different classes of known filters always entail the release of one of the following desired properties : zero inter-symbol interference, phase linearity (at least at sending) and the pair of matched filters.

Often, in communication systems, it becomes necessary to accept that the filtering introduces a non-zero ISI. This however sometimes raises problems, for

example in the case of modulations with very large numbers of states (64, 256, ...).

It is an object of the invention especially to overcome these drawbacks of the prior art.

More specifically, it is an object of the invention to provide a method for the making of Nyquist filters with null inter-symbol interference that also meets the desired conditions of frequency selectivity and phase linearity in a matched pair configuration.

Another aim of the invention is to provide a method such as this that enables easy practical implementation of the filters obtained and takes account of certain practical constraints such as the effects of the quantification of the coefficients. More specifically, a goal of the invention is to provide a method to ensure the maintenance of the property of cancellation of inter-symbol interference after this quantification.

Another aim of the invention is to provide a method of this kind that can be implemented by means of a cascade structure, especially in lattice form.

These aims as well as others that shall appear hereinafter are achieved according to the invention by means of a method for the making of a digital Nyquist filter with null inter-symbol interference designed to process a physical signal transmitted between a sender and a receiver through a transmission channel. This filter is an Nth order $P(z) = F^2(z)$ symmetrical filter implementing an oversampling factor M=4 and forming a matched pair comprising a sending filter and a reception filter whose polyphase breakdown of F(z) can be written as follows:

$$F(z) = F_0(z^4) + z^{-1}F_1(z^4) + z^{-2}F_2(z^4) + z^{-3}F_3(z^4).$$

According to the invention, N is chosen to be different from 4n, n being an integer and the coefficients of the polyphase breakdown of F(z) are such that:

- If N=4n+1, $F_1(z)\hat{F}_1(z) + z^{-1}F_2(z)\hat{F}_2(z) = \gamma z^{-n}$
- If N=4n+2, $2F_0(z)\hat{F}_0(z) + F_1^2(z) + z^{-1}F_3^2(z) = \gamma z^{-n}$
- If N=4n+3, $F_0(z)\hat{F}_0(z) + F_1(z)\hat{F}_1(z) = \gamma z^{-n}$

$\hat{F}$ being the mirror symmetry of F and γ being a non-null constant.

Thus, the method of the invention, by construction, ensures that the ISI is perfectly null.

Preferably, N is equal to 4n+3 or 4n+1 and:

said sending filter performs an interpolation by a factor M = 4 and has a circuit arrangement corresponding to a polyphase breakdown known as the type II breakdown, such that:

$$F(z)=\left[z^{-3}\ z^{-2}\ z^{-1}\ 1\right]\begin{bmatrix}\hat{F}_0(z^4)\\ \hat{F}_1(z^4)\\ F_1(z^4)\\ F_0(z^4)\end{bmatrix}$$

and said reception filter performs a decimation by a factor M = 4 and has a circuit arrangement corresponding to a polyphase breakdown known as the type I breakdown, such that:

$$F(z)=\left[F_0(z^4)\ F_1(z^4)\ \hat{F}_1(z^4)\ \hat{F}_0(z^4)\right]\begin{bmatrix}1\\ z^{-1}\\ z^{-2}\\ z^{-3}\end{bmatrix}$$

The filters thus obtained are simple to make and implement.

Advantageously, in said sending filter, there is performed a filtering step followed by a step of interpolation by a factor M = 4. Similarly, in said reception filter, advantageously a step of decimation by a factor M = 4 is performed, followed by a filtering step.

This construction (permutated structure) reduces the rate of the operations by a factor 4.

According to a preferred embodiment of the invention, said sending filter and/or said reception filter has a structure in the form of at least one lattice.

Indeed, in this form of circuit arrangement, the constraint of perfect reconstruction is structurally integrated.

Advantageously, said sending filter and said reception filter are each constituted by a pair of polyphase components respectively given by the following equations:

$$\begin{bmatrix} F_0 \\ F_1 \end{bmatrix} = gA(\alpha_n)\Lambda(z)A(\alpha_{n-1})\ldots\Lambda(z)A(\alpha_0)\begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} -\hat{F}_1 \\ \hat{F}_0 \end{bmatrix} = gA(\alpha_n)\Lambda(z)A(\alpha_{n-1})\ldots\Lambda(z)A(\alpha_0)\begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

with : $$A(\alpha) = \begin{bmatrix} 1 & \alpha \\ -\alpha & 1 \end{bmatrix} \quad \text{et} \quad \Lambda(z) = \begin{bmatrix} 1 & 0 \\ 0 & z^{-1} \end{bmatrix}$$

where g is a non-null constant of standardization and α_i are real coefficients.

According to one particular embodiment of the invention, the method implements a two-lattice structure. According to another embodiment, it may implement a single-lattice structure working at a double frequency.

The invention also relates of course to filtering devices obtained by means of the above-described method. These are therefore devices for the filtering of Nyquist digital signals with null inter-symbol interference designed to process a physical signal transmitted between a sender and a receiver through a transmission channel, based on an Nth order $P(z) = F^2(z)$ symmetrical filter implementing an oversampling factor $M = 4$ and forming a matched pair comprising a sending filter and a reception filter, the polyphase breakdown of F(z) of this symmetrical filter being written as follows:

$$F(z) = F_0(z^4) + z^{-1}F_1(z^4) + z^{-2}F_2(z^4) + z^{-3}F_3(z^4).$$

According to the invention, N is different from 4n, n being an integer, and:

- If N=4n+1, $\quad F_1(z)\hat{F}_1(z) + z^{-1}F_2(z)\hat{F}_2(z) = \gamma z^{-n}$
- If N=4n+2, $\quad 2F_0(z)\hat{F}_0(z) + F_1^2(z) + z^{-1}F_3^2(z) = \gamma z^{-n}$
- If N=4n+3, $\quad F_0(z)\hat{F}_0(z) + F_1(z)\hat{F}_1(z) = \gamma z^{-n}$

$\hat{F}$ being the mirror symmetry of F and γ being a non-null constant.

Other features and advantages of the invention shall appear more clearly from the following description of a preferred embodiment given by way of a simple, illustrative and non-restrictive example and from the appended drawings, of which:

- Figure 1, already commented upon in the introduction, provides a diagrammatic illustration of the digital part of a communications system;
- Figure 2, commented on in Appendix 1, represents a typical frequency specification of a sending filter;
- Figure 3, commented upon in Appendix 2, gives a general view of a bank of filters with two sub-bands;
- Figure 4 illustrates an embodiment of the analysis part of a bank of para-unitary filters with two sub-bands of Figure 3 in lattice form:
- Figure 5 recalls the structure of a sending/reception filtering system in the linear and para-unity case commented on hereinafter;
- Figures 6a and 6b give a view, for a 4n+3 order filter of two forms of circuit arrangement of the sending filter of Figure 5, using a polyphase breakdown respectively in a direct structure and a permutated structure;
- Figures 7a and 7b provide an illustration, in the case of the 4n+3 order, of the reception filter of Figure 5 also in the context of polyphase breakdown respectively according to a direct structure and a permutated structure;
- Figures 8a and 8b show a lattice block of the sending/reception filters of Figures 6a, 6b, 7a and 7b respectively in the form of a direct lattice and a lattice with inverted outputs;
- Figure 9 gives a detailed view of a 4n + 3 order Nyquist root sending filter with two lattices implementing the inverted lattices of Figure 8b;
- Figure 10 shows a 4n+3 order Nyquist reception filter implementing four lattices;
- Figure 10b shows a reception filter implementing a direct lattice according to Figure 8a (see Appendix 4);
- Figure 11 recalls the equations used in the case of the inverted lattice of Figure 9;

- Figure 12 shows another embodiment of the sending filter implementing a single lattice and working at a double rate;
- Figure 13 shows an exemplary response in optimized frequencies of a 43rd order Nyquist root filter according to the invention;
- Figure 14 illustrates the embodiment of a block Z in the case of a 4n+2 order filter as discussed in Appendix 3;
- Figure 15 shows the circuit arrangement block of the matrix P^T also in the context of the embodiment of Appendix 3;
- Figure 16 shows the full circuit arrangement diagram of a 4-input and 4-output block for the 4n+2 order referenced Ma and discussed in Appendix 3;
- Figure 17 shows an embodiment of a half Nyquist sending filter for the 4n+2 order;
- Figures 18 and 19 (block N of Figure 18) illustrate a half Nyquist reception embodiment for the 4n+2 order discussed in Appendix 3;
- Figures 20 and 21 respectively show the sending and reception Nyquist root filters for the 4n+1 order (or 4n+5 order) as discussed in Appendix 5;
- Figures 22 and 23 illustrate the results obtained by means of the mode of generic synthesis commented upon in Appendix 6.

As mentioned already, the method of the invention can be used to make filters with M = 4 verifying the totality of the following criteria:

- ISI null by construction;
- linear phase filters;
- sending and reception filters forming a matched pair.

Appendix 2, after developing the analogy between a Nyquist pair with M = 4 and a bank of filters orthogonal to two sub-bands; shows that the two situations may occur according to the order of the filter. More specifically, it is shown that there is no solution for the N = 4n order filters and that the cases N = 4n + 1 and N = 4n + 3 are favored (theorem 2) in that they produce similar polyphase components. The condition for obtaining null inter-symbol interference is specified in the theorem 1.

The case of the 4n + 3 filters is described hereinafter. It proves to be the case used to obtain the simplest embodiment. The case 4n + 2 is processed in Appendix 3 and the case 4n + 1 (or 4n + 5) in Appendix 5.

As a rule, the basic diagram implemented in shown in Figure 5. As compared with the initial general diagram of Figure 1, it may be noted that $F_T(z)$ 122 and $F_R(z)$ 151 are replaced by F(z). This is indeed a linear phase matched pair.

We must also note the introduction of a delay line z-r51 where r is a positive integer value dependent on the order, which in all cases makes it possible to obtain a null ISI with a given delay. The multiplier factor g 52 is used to ensure that, except for the delay, the output is quite identical to the input.

In the case where N = 4n + 3, because of the relationships between the different polyphase components F(z), the expression (20) of Appendix 2 can be rewritten as follows:

$$F(z) = F_0(z^4) + z^{-1}F_1(z^4) + z^{-2}\hat{F}_1(z^4) + z^{-3}\hat{F}_0(z^4) \quad (31)$$

It is known that, apart from the delay, the sending and reception filters are identical. However, if it is sought to draw the best advantage of the multi-rate processing operations, namely interpolation at sending and decimation at reception, the structural drawings will not be exactly the same.

At sending, where an interpolation by a factor M = 4 is made, the forms of circuit arrangement shown in Figures 6a and 6b correspond to those known as type II polyphase breakdown [4] (the references cited are assembled in Appendix 7), given in the case of the invention by:

$$F(z) = \begin{bmatrix} z^{-3} & z^{-2} & z^{-1} & 1 \end{bmatrix} \begin{bmatrix} \hat{F}_0(z^4) \\ \hat{F}_1(z^4) \\ F_1(z^4) \\ F_0(z^4) \end{bmatrix} \quad (32)$$

In one of the depictions (Figure 6b), it is possible to note the permutation of the expansion (61) and filtering (62) operations used to carry out an equivalent processing by working at a rate four times lower than in the case of the initial diagram (Figure 6a).

In the reception part, the filtering is associated with a decimation operation for which a more appropriate writing of F(z) is that of what is called the type I [4] polyphase decomposition which in the present case can be written as follows:

$$F(z) = \left[F_0(z^4)\ F_1(z^4)\ \hat{F}_1(z^4)\ \hat{F}_0(z^4)\right]\begin{bmatrix}1\\ z^{-1}\\ z^{-2}\\ z^{-3}\end{bmatrix} \qquad (33)$$

It will be noted that, with reference to Figure 5, the time limit is such that r = 1.

Two variants of the circuit arrangement of this equation are given in Figures 7a and 7b (symmetrically with Figures 6a and 6b).

The analogy between the 4n+3 order filter F(z) and the 2n +1 order bank of filters described by the theorem 3 (Appendix 2) is used also to rewrite the equation in the form:

$$\begin{bmatrix}F_0(z) & -\hat{F}_1(z)\\ F_1(z) & \hat{F}_0(z)\end{bmatrix} = gA(\alpha_n)\Lambda(z)A(\alpha_{n-1})\ldots\Lambda(z)A(\alpha_0) \qquad (34)$$

The two pairs of polyphase components which are mirrors of each other are therefore deduced from each other by the following two equations:

$$\begin{bmatrix}F_0\\ F_1\end{bmatrix} = gA(\alpha_n)\Lambda(z)A(\alpha_{n-1})\ldots\Lambda(z)A(\alpha_0)\begin{bmatrix}1\\ 0\end{bmatrix} \qquad (35)$$

and

$$\begin{bmatrix}-\hat{F}_1\\ \hat{F}_0\end{bmatrix} = gA(\alpha_n)\Lambda(z)A(\alpha_{n-1})\ldots\Lambda(z)A(\alpha_0)\begin{bmatrix}0\\ 1\end{bmatrix} \qquad (36)$$

In each of these cases, the lattice structure shown in Figures 8a and 8b is the same.

The two variants, with direct or inverted outputs of Figure 8, are then used to set up the sending and reception filters according to the diagrams respectively shown in Figures 9 and 10. It will be noted that the symbols at output of the filters indicate only which are the active filters.

By using the polyphase decomposition of the input signal, it is then possible to determine an equivalent diagram with only one lattice which however, in this case, is constrained to work at the double frequency.

In the diagram proposed, the inverted lattice is presented, it being known of course that the direct lattice leads to a diagram of the same type. Figure 11 recalls the basic equations of this lattice.

The output signal of the sending filter is written as follows:

$$Y(z)=z^{-1}\left[F_0(z^4)+z^{-1}F_1(z^4)+z^{-2}\hat{F}_1(z^4)+z^{-3}\hat{F}_0(z^4)\right]X(z^4) \qquad (37),$$

which can be rewritten as follows:

$$\begin{aligned}Y(z)&=z^{-1}\left[F_0(z^4)+z^{-2}\hat{F}_1(z^4)\right]X(z^4)+z^{-2}\left[F_1(z^4)+z^{-2}\hat{F}_0(z^4)\right]X(z^4)\\&=\left[Y_0(z^2)+z^{-1}Y_1(z^2)\right]z^{-1}\end{aligned} \qquad (38)$$

A division by 2 of the expansion factor ($\uparrow 4 \equiv \uparrow 2 \uparrow 2$), and then a permutation of the expansion of 2 with the filtering produces filtered outputs such that:

$$\begin{aligned}Y_0(z)&=\left[F_0(z^2)X(z^2)+z^{-1}\hat{F}_1(z^2)X(z^2)\right]\\Y_1(z)&=\left[F_1(z^2)X(z^2)+z^{-1}\hat{F}_0(z^2)X(z^2)\right]\end{aligned} \qquad (39).$$

In $Y_1(z)$, it is possible to recognize the higher output of the inverted lattice working at double rate and, plus or minus the sign of the term as a factor of z^{-1}, in $Y_0(z)$ it is possible to recognize the lower output. To recover exactly the same expression, it is therefore enough to multiply this output of the inverted lattice by $(-1)^n$.

One embodiment based on this principle is shown in Figure 12.

For the reception part, it is possible to consider two cases:

- the general case, with four lattices, which will need all the desired properties (Figure 10);
- the simplified diagram for which only the null ISI and the phase linearity at sending are guaranteed (Figure 10b).

This particular aspect is discussed in Appendix 4.

In the case of Figure 9, the system works at the lowest frequency. The cost of each filter corresponds to a lattice structure with n+1 cells plus that of a sign

inversion. To obtain a unit gain, a multiplication by the factor g generally must be accounted for in the diagram of the reception filter.

The computations for each elementary filter correspond to the two multipliers and adders. However, as in the case of each lattice, one of the inputs is at zero, and a multiplier and an adder can therefore be eliminated. The cost is therefore one multiplier and one adder for this first cell and double for the following, namely (2n + 1) multipliers and (2n +1) adders. For each filter, we will therefore have (4n + 2) multipliers and (4n + 2) adders. At the lowest frequency, we will therefore have to make (4n + 2) MPU and (4n +2) APU.

For the reception part, in the case of the simplified diagram illustrated by Figure 10b, the operational complexity is that of a single lattice rather than that of three adders, the multiplier by g and the inverter. The computations performed at the lowest rate can be estimated at (2n + 3) MPU, (2n + 5) APU and one inversion.

In the general case illustrated by Figure 10, at the lowest frequency, the complexity is (8n + 4) MPU and APU and one inversion.

The method of synthesis is performed for example in three steps, illustrated here below by the computation of a 43rd order filter.

- Step 1: Using the synthesis tools [3], it is possible to obtain a first set of transversal coefficients that are close to null ISI. These are the initial coefficients of Table 1.
- Step 2: By identification, we can compute the corresponding initial lattice coefficients given in the Table 2. Since the initial set of transversal coefficients is not exactly at null ISI, a new set of transversal coefficients is thus obtained known as reconstructed coefficients (cf. Table 1).
- Step 3: A local optimizing of the initial lattice coefficients then provides for slight improvement in the frequency response of the filter. Tables 1 and 2 give the values of these lattice and transversal coefficients after this optimization (Tables 1 and 2).

Table 1 : Transversal coefficients of the 43rd degree symmetrical filters

i	Initial	Reconstructed	Optimizal
0	0.006302365568	0.006302365568	0.00374315552336009
1	-0.01335763186	-0.01335763186	-0.00764365688411938
2	-0.003586256877	0.000139948824594686	0.000256865997947587
3	0.003281590529	0.000296616382849522	0.00052452951560661
4	0.008254154585	0.00825445268157875	0.00210146572675043
5	0.008753151633	0.00875251982827294	0.00961384935212083
6	0.002954708412	-0.000818038274240153	-0.000565430770503378
7	-0.006948650815	-0.00231664836261536	-0.00210883853472157
8	-0.01474214438	-0.0147620230725521	-0.00994253905900314
9	-0.01369718742	-0.0136538137768085	-0.0139493173848354
10	-0.001682000235	0.00199238622180159	0.00140539009490696
11	0.01571460254	0.00939101589973353	0.00785652590927169
12	0.026870884	0.0269017853424573	0.0234276140713132
13	0.02112355083	0.0209715158594175	0.0216783222304717
14	-0.003340088297	-0.0032097877874163	-0.00339966101869637
15	-0.03511243314	-0.0308000965511264	-0.0274417538899884
16	-0.05300379917	-0.0530202725288858	-0.0511658629696837
17	-0.0364661812	-0.0360408764445379	-0.0377795925012697
18	0.0221698768	0.0140041435375816	0.0161657062857809
19	0.1105730906	0.116698398397514	0.109490464868349
20	0.1994492114	0.199658215352476	0.199471282748299
21	0.2550483644	0.253344105750997	0.257329314844394

Table 2 Coefficients of the lattice structures

	Initial	Optimizal
$\tilde{g}_0$	0.006302365568	0.003743155523
α_1	2.1194631945539	2.04203561311232
α_2	- 0.7583059650965	- 0.71854479175763
α_3	0.6811397085317	0.73802350460893
α_4	- 0.4637577935126	- 0.55360950120952
α_5	0.4470434662649	0.51415010432349
α_6	- 3.7431288142180	- 5.34790460147966
α_7	- 1.5196264288798	- 1.52189568702330
α_8	1.2422617481334	1.44910973460487
α_9	- 0.5612421724760	- 0.63596605846521
α_{10}	0.1588040486787	0.18869460160260
α_{11}	- 0.0222057611677	- 0.06862284945003

The frequency response for the 43rd-order filter computed in this way is shown in Figure 13.

This is a first result that may be further refined. Furthermore, of course, other computation techniques may be considered for different technical orders and constraints.

It will be recalled especially that the case $N = 4n + 2$ is presented in the Appendix 3. Furthermore, Appendix 5 discusses the case $N = 4n + 1$ (or $4n + 5$).

Details of the assessment of the complexity of the reception filters and a method of generic synthesis are developed in Appendix 6. It will be noted that this method of synthesis can be applied to all three types of approaches described here above.